# APPENDIX D <br> GUIDE TO BENCHMARKING OPERATIONS PERFORMANCE MEASURES 

## Analysis of Sample Size

## Contents

Statistical Patterns of Traffic Data and Sample Size Estimation -
A paper prepared from the result of a statistical analysis of traffic sensor data to determine appropriate sample sizes for performance measurement

# STATISTICAL PATTERNS OF TRAFFIC DATA AND SAMPLE SIZE ESTIMATION 

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#### Abstract

Sample size estimation is fundamental to traffic engineering analysis. An iterative procedure using standard deviation is the most reliable method of sample size estimation, but practitioners find it cumbersome. Using a $t$-statistic in the calculation further adds to the complexity due to dependence on sample size. Previous attempts have been made to simplify this process and render it a one-step exercise. The ITE Manual of Traffic Engineering Studies provides representative values of standard deviation for spot speeds on roadways classified by AADT volumes, which have been found to be inadequate for arterials and low and heavy traffic conditions on all roadways. Other simplified methods borrow from quality control theory and use average sample range instead of standard deviation in the calculation. Such methods assume a simple relationship between the two, which is not necessarily the case. Sample size estimation using standard deviation remains the most reliable method and is preferred whenever reliable estimates of standard deviation can be obtained. This study estimated standard deviations of speed and volume counts from inherent statistical patterns of traffic data; patterns which have been observed across different locations and aggregation levels of time. This study found a consistent U-shaped relationship between the standard deviation of speed and hourly volume for a wide variety of conditions. This relation was based on a large number of observations at multiple sites, which justifies the use of a $z$ statistic in the final calculation of sample size. A different consistent pattern for variability of volume count was also observed.


## INTRODUCTION

Sample size estimation is fundamental to traffic engineering analysis. Typically, sample size estimation is based on the standard deviation of the performance measure under study. If the standard deviation for the given measure is known, required sample size can be calculated in a single step using a simple formula. Otherwise, an iterative procedure using estimates of standard deviation is used as the most reliable method for sample size calculation. In this procedure, an initial number of data points for the given measure are collected and the standard deviation is estimated. Using this standard deviation, required sample size is calculated for a desired accuracy at a given confidence level. If the calculated sample size is less than the initial number of data points, then the data taken initially is sufficient to estimate the performance measure with the given accuracy and confidence level. Otherwise, the number of data points dictated by the calculation is collected, standard deviation is estimated for this new sample, and the sample size calculation is once again performed. This procedure is repeated until convergence occurs and a value for sample size is obtained. This procedure usually provides a sample size that is greater than the minimum requirement.

Apart from the iterative nature of the above procedure, there are other associated complexities from a statistical point of view. If the number of data points that are used for calculations are greater than or equal to thirty, it can be assumed that the estimator of the performance measure (which is usually arithmetic mean) is normally distributed, and hence the $z$-statistic for the given confidence level can be used in the formula for sample size calculation. Otherwise, Student's $t$-statistic, which also incorporates the sample size and confidence level, should be used in the formula.

Naturally, traffic engineers and field practitioners find the sample size calculation process cumbersome. In the past, attempts have been made to simplify this process and even render it a one-step exercise. Some literature suggests representative standard deviation values for spot speeds on different roadways classified by average annual daily traffic (AADT) volumes. Although such figures are in consonance with the findings of this study, they are based on rather aggregate characteristics of traffic data and may be inadequate for operations and intelligent transportation system (ITS) applications. Another method present in literature is borrowed from quality control theory and makes use of the average range of a sample instead of its standard deviation. This method assumes a simple relationship between these two quantities, which may not necessarily hold true for all datasets.

Sample size calculation using standard deviation remains the fundamental and most reliable method. Theoretically, a value for sample size cannot be given a priori, and an iterative procedure remains which must make use of the local data. However, there exist patterns for standard deviations of speed and volume counts versus per lane hourly volume, which are derived from disaggregate traffic data (5-minute interval detector data). Values obtained from these patterns can be used for locations where data cannot be collected due to technical or economic limitations. These patterns hold at different aggregation levels (15minute and 1-hour) and across different locations. Values of standard deviation obtained in this manner are better suited for traffic operations and real-time applications.

## SURVEY OF SAMPLE SIZE CALCULATION METHODS

The minimum sample size required to estimate a variable with an accuracy of $\pm \varepsilon$ units at a certain confidence level is given by:

$$
n=\left(\frac{t \times s}{\varepsilon}\right)^{2}
$$

[Eqn. 1]
$s=$ Sample standard deviation
$t=$ Student's $t$-statistic for given confidence level and degree of freedom
If the variable is normally distributed, or a large sample $(\geq 30)$ is available to calculate the standard deviation, the $z$-statistic can be used instead of the $t$-statistic. This is the fundamental equation for calculating sample size, but it may not be the simplest method. There are other methods available which approximate the above equation using certain assumptions and are easier to work with. However, it should be kept in mind that the above equation remains the most reliable way of calculating sample size and should be preferred whenever a reliable estimate of standard deviation can be obtained. This is where the emphasis of the present work lies.

In literature, sample size guidelines for traffic engineering studies are primarily available for spot speed and travel time. Fundamentally, the process for estimating required sample size should not be different for these two measures when data for each is available. However, whereas the Manual of Traffic Engineering Studies and the Manual of Transportation Engineering Studies use standard deviation to calculate sample size for spot speed, they adopt an entirely different approach for travel time and delay (1,2). In these manuals, sample size guidelines for travel time and delay are provided in terms of travel speed data rather than travel time. Average range in travel speed, and not its standard deviation, determines the required number of runs (or sample size). This method utilizes techniques of quality control theory, where $3 \sigma(99.7 \%$ confidence level) control limits for the mean are determined using the average range in data (3).

The average range, $\bar{R}$, is calculated by taking the average over absolute differences between sequential data points (i.e., second minus first, third minus second, etc.). It is not clear why significance has been attached to the order in which data was collected. The formulation for sample size is given as:

$$
\begin{equation*}
n=\left(\frac{z \times \bar{R}}{d \times \varepsilon}\right)^{2} \tag{Eqn.2}
\end{equation*}
$$

Where $z$ is the $z$-statistic for a given confidence level and $d$ is the ratio of $\bar{R}$ to standard deviation, $\sigma$, which can be obtained from existing literature (3). Alternatively, the required sample size at a $99.7 \%$ confidence level $\left(N_{l}\right)$ can also be obtained directly from existing literature (3) for a given $\bar{R}$. The sample size at some other confidence level ( $N_{2}$ ) can be calculated using:

$$
\begin{equation*}
N_{2} / N_{1}=\left(z_{2} / z_{1}\right)^{2} \tag{Eqn.3}
\end{equation*}
$$

Where $z_{1}$ and $z_{2}$ are normal deviates at corresponding confidence levels.
Validity of the normal distribution assumption is critical in using Eqn. 3 - and this assumption is questionable, because in most practical cases the recommended sample size is less than thirty $(1,2,3)$. Overall, this method greatly simplifies the iterative procedure of sample size calculation and is easier to work with due to the fact that calculating the average range is simpler than calculating the standard deviation of the sample. It makes the assumption, however, that a simple ratio between average range and standard deviation exists, which does not necessarily hold. Li et al. (4) have found this method to be the least satisfactory when compared with "hybrid" and "modified" methods, which are explained below.

Quiroga and Bullock (5) noted that the aforementioned method of calculating $\bar{R}$ is biased since the moving ranges are correlated through their common speed and its value tends to be lower than the true value. A "hybrid" formulation is put forth, which retains the above definition of $\bar{R}$ because of its intuitive and simple-to-use nature, and compensates it by changing the $z$-statistic in Eqn. 2 to the $t$-statistic (since $t>z$, and also because sample size is almost always less than 30). The authors also note that there are systematic numerical errors in the sample size tables provided in (2) and (3).

Li et al. (4) reverted to the use of standard deviation from average range for accuracy. Use of the $z$-statistic in place of the $t$-statistic is also suggested for ease of calculation, and an adjustment factor is introduced to balance the discrepancy. The "modified" equation is:

$$
\begin{equation*}
n=\left(\frac{z \times \sigma}{\varepsilon}\right)^{2}+\varepsilon_{n} \tag{Eqn.4}
\end{equation*}
$$

Based on numerical results, the value of $\varepsilon_{n}$ is recommended as 2,3 and 4 for confidence levels $90 \%, 95 \%$ and $99 \%$, respectively. Another work suggested a constant value of 2 for $\varepsilon_{n}$ in situations with less than 30 observations (6).

Oppenlander (7) estimated standard deviations of spot speeds to describe the variability for various roadway types classified by AADT volume. Average standard deviations for the different roadways ranged from 4.16 to 5.31 mph , and a value of 5.0 mph was suggested for any highway in any traffic area (1,2,7). With better surveillance tools available today to collect very disaggregate data, similar analysis has been performed in this paper to find a trend between the standard deviation and hourly traffic volume.

Within the standard deviation-based methods, there are two different formulations depending upon the specification of permitted error. When permitted errors are specified in absolute values, Eqn. 1 is used. If permitted errors are specified in percent values ( $\alpha$ ), then a different formulation based on the previous one is used:

$$
\begin{equation*}
n=\left(\frac{t \times s}{\varepsilon}\right)^{2}=\left(\frac{t \times s}{\alpha / 100 \times \bar{x}}\right)^{2}=\left(\frac{t \times c_{v}}{\alpha / 100}\right)^{2} \tag{Eqn.5}
\end{equation*}
$$

$\bar{x}=$ Sample mean
$c_{v}=$ Coefficient of variation, $(s / \bar{x})$
In such cases, guidelines are provided in terms of representative values of the coefficient of variation for different facility types. NCHRP Report 398 - Quantifying Congestion - used a large data set of travel time for arterials and freeways to calculate the average and the $85^{\text {th }}$ percentile values of $c_{v}$ for sample size calculation purposes (6). Such results are very useful for traffic studies. However, traffic data for different conditions (peak hours, off-peak hours, etc.), which are likely to have different variability, were mixed together in the calculation.

Turner and Holdener (8) have performed more focused analysis by calculating the $85^{\text {th }}$ percentile of $c_{v}$ for travel time using 21 weekdays' worth of only peak-hour data for Houston. Such granular level analyses are better suited for real-time applications. Moreover, attempts to estimate population $c_{v}$ using large data sets in (6) and (8) have better justification for using the $z$-statistic in the sample size calculation. In the present work, similar analyses have been performed for speed and volume (subsequent work is ongoing for travel time) using 24-hour data for several weekdays, and variability is calculated for different volume levels.

## STATISTICAL PATTERNS OF TRAFFIC DATA

In order to establish any possible trend in the variability of speed and volume counts with traffic volume levels, traffic detector data was obtained for several locations. Speed, volume and occupancy data for arterials and freeways were available for 15 -minute and 5-minute intervals, respectively.

The $15-$ minute and 5 -minute average speed and volume count data were grouped into bins containing a range of 200 vehicles per hour per lane (veh/hr/ln), based on the hourly volume they belonged to (i.e., Group 1: $0-200 \mathrm{veh} / \mathrm{hr} / \mathrm{ln}$; Group 2: $200-400 \mathrm{veh} / \mathrm{hr} / \mathrm{ln}$, etc.). For a given location, standard deviations of speed and volume counts were calculated for each volume group using at least five weekdays' worth of data. Before dividing the data into volume groups, hourly volume per lane for a particular hour was calculated by adding up 15-minute/5-minute volume counts for that hour and then dividing the sum by the number of lanes. For simplicity's sake, in order to calculate hourly volume, time intervals were always chosen as $8: 00 \mathrm{AM}-8: 59 \mathrm{AM}, 2: 00 \mathrm{AM}-2: 59 \mathrm{AM}$, etc. Hourly volume was never calculated for such time intervals as $8: 15 \mathrm{AM}-9: 14 \mathrm{AM}$ or $2: 45 \mathrm{AM}-3: 44 \mathrm{AM}$.

In the case where two or more 15 -minute entries were missing from the arterial data for a given hour, all other entries for that particular hour were discarded from analysis. If only one 15 -minute entry was missing in a given hour, hourly volume was calculated by extrapolation. Since only hourly volume range, not actual volume, is used in determining the appropriate volume group, the overall analysis is expected to be insensitive to this adjustment. Additionally, the cases for a single missing entry were observed only between 12:00AM 12:59AM on each day, and the volume for this hour was small and always fell into the first group (i.e., $0-200 \mathrm{veh} / \mathrm{hr} / \mathrm{ln}$ ).

In terms of freeway data, if less than nine 5-minute entries were present for a given hour, the hourly data was not used in the analysis. Otherwise, extrapolated hourly volume was used for classification into volume bins.

## Arterial

Data for arterials was obtained for several locations in Northern Virginia along both directions of Reston Parkway and Route 7. Individual lane detector data at these locations was also available. These locations were near intersections, but were sufficiently upstream of the intersections such that they can be assumed to be approximately representative of the midblock traffic conditions.

For each location, standard deviations of 15-minute average speed and volume counts were calculated for each volume group using data for five weekdays. These calculations were performed for each lane individually and also collectively for a given location by pooling lane-wise data. Due to the fact that individual lane-wise results and aggregated location-wise results were similar to one another, only location-wise results are presented below.

## Speed

Standard deviation values of speed for different locations are presented in Table 1. Some of the locations had very different standard deviation values for certain volume groups when compared to other locations, and such outlier entries are italicized and highlighted in the table.

TABLE 1 Standard Deviations of 15-minute Average Speed on Arterials

| Location | Dulles Toll (East) | Baron Cameron Ave. | Sunset Hills Rd. | Sunrise <br> Valley Dr. | Baron Cameron Ave. | Bluemont <br> Way | S Lakes Dr. | Levinsville Rd. | Spectrum <br> Center | Bluemont Way |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Roadway | Reston (SB) | Reston (SB) | Reston (SB) | Reston (NB) | Reston (NB) | Reston (NB) | Reston (SB) | Rt. 7 (WB) | Reston (NB) | Reston (SB) |
| Volume (veh/hr/ln) | (mph) | (mph) | (mph) | (mph) | (mph) | (mph) | (mph) | (mph) | (mph) | (mph) |
| 0-200 | 7.1 | 8.7 | 8.8 | 9.4 | 9.7 | 10.2 | 10.3 | 10.7 | 10.9 | 12.3 |
| 200-400 | 3.2 | 4.5 | 3.1 | 2.7 | 5.5 | 3.8 | 3.9 | 5.4 | 3.7 | 6.0 |
| 400-600 | 2.3 | 3.1 | 6.4 | 5.0 | 5.2 | 3.7 | 3.1 | 6.4 | 4.8 | 5.9 |
| 600-800 | 2.2 |  | 6.4 | 7.9 |  | 2.8 | 3.0 | 5.6 | 5.5 | 3.2 |
| 800-1000 | 4.8 |  |  | 5.6 |  | 2.4 | 3.7 | 7.5 | 7.5 |  |
| 1000-1200 | 3.9 |  |  | 5.5 |  | 1.4 | 3.3 | 4.1 |  |  |
| 1200-1400 |  |  |  |  |  |  |  | 3.2 |  |  |
| 1400-1600 |  |  |  |  |  |  |  |  |  |  |
| 1600-1800 |  |  |  |  |  |  |  |  |  |  |


| Location | Sunrise <br> Valley Dr. | Glade Dr. | Sunset Hills <br> Rd. | Glade Dr. | New <br> Dominion Pkwy. | Delta Glenn Ct. | New <br> Dominion <br> Pkwy. | Delta <br> Glenn Ct. | Spectrum Center | S Lakes Dr. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Roadway | Reston (SB) | Reston (SB) | Reston (NB) | Reston (NB) | Reston (NB) | Rt. 7 (WB) | Reston (SB) | Rt. 7 (EB) | Reston (SB) | Reston (NB) |
| Volume (veh/hr/ln) | (mph) | (mph) | (mph) | (mph) | (mph) | (mph) | (mph) | (mph) | (mph) | (mph) |
| 0-200 | 13.3 | 13.5 | 13.9 | 14.6 | 14.7 | 15.1 | 15.1 | 16.1 | 16.2 | 19.4 |
| 200-400 | 4.4 | 4.0 | 15.8 | 3.8 | 7.3 | 7.5 | 9.2 | 8.4 | 17.5 | 7.8 |
| 400-600 | 5.6 | 3.3 | 14.3 | 3.1 | 7.5 | 11.1 | 9.2 | 6.0 | 23.5 | 9.1 |
| 600-800 | 7.5 | 2.9 | 9.5 | 3.2 | 7.5 | 11.1 | 6.9 | 8.9 | 27.9 | 16.6 |
| 800-1000 | 11.6 | 7.0 | 10.8 | 4.4 | 3.6 | 3.1 | 2.4 | 10.7 |  |  |
| 1000-1200 | 13.2 | 9.7 | 16.5 | 3.2 |  | 5.6 |  | 11.8 |  |  |
| 1200-1400 |  |  | 13.0 |  |  | 10.3 |  | 17.7 |  |  |
| 1400-1600 |  |  |  |  |  | 10.0 |  | 13.4 |  |  |
| 1600-1800 |  |  |  |  |  |  |  | 13.8 |  |  |

Several commonly-used summary statistics (average, median, $70^{\text {th }}$ percentile and $80^{\text {th }}$ percentile) for standard deviation values across locations were calculated. These statistics, calculated for both the complete set of all data points and for the data without outlier points, are presented in Figure 1. The general trend between the standard deviation of 15 -minute average speed and per lane hourly volume can be described as follows:

With volumes less than $200 \mathrm{veh} / \mathrm{hr} / \mathrm{ln}$, the standard deviation is characterized by very high values. For increasing volume ranges, however, the standard deviation value decreases approximately by half and remains roughly stable throughout the 200 $1000 \mathrm{veh} / \mathrm{hr} / \mathrm{ln}$ volume ranges. For volumes greater than $1000 \mathrm{veh} / \mathrm{hr} / \mathrm{ln}$, the standard deviation value once again increases. This data showcases a U-shaped relation between standard deviation and volume.

Both of the below plots (Figures 1 (a) and (b)) corroborate the general U-shaped relation, and also allude to its robustness in the presence of outlier data. Based on the available results, the authors have suggested typical values of standard deviations, which are displayed in Figure 1 (b). End users of these results may choose a more or less conservative estimate of the standard deviation depending upon their requirements.

(a) Variability of speed versus hourly volume plotted using all data.

(b) Variability of speed versus hourly volume plotted after discarding outlier data.

FIGURE 1 Variability of arterial speed with hourly volume.

## Volume

Similarly to the aforementioned speed calculations, standard deviations of 15 -minute volume counts for different volume groups were calculated. However, the standard deviation of 15minute volume counts belonging to two different hourly volume ranges cannot be compared directly. A standard deviation of $30 \mathrm{veh} / 15-\mathrm{min} / \mathrm{ln}$ when the traffic flow is $200 \mathrm{veh} / \mathrm{hr} / \mathrm{ln}$ does not represent the same amount of variability as a standard deviation of $30 \mathrm{veh} / 15-\mathrm{min} / \mathrm{ln}$ when the traffic flow is $1800 \mathrm{veh} / \mathrm{hr} / \mathrm{ln}$ even though both traffic flows have the same standard deviation values. Therefore, standard deviation values of 15 -minute volume counts normalized by the mean value (i.e., coefficient of variation) are deemed suitable measures to discern a trend with hourly volume. Since coefficient of variation, and not standard deviation, is chosen for volume counts, the permitted error for sample size calculation should be specified as a percentage (see Eqn. 5).

Further, volume is a cumulative measure, unlike speed. If speed data is collected for 5-minutes, a certain number of observations will be gathered in order to make a statement to the effect that the observed speed is $x \mathrm{mph}$ with a certain level of confidence. If data is collected over twice the initial duration (i.e., 10 minutes) speed will still be described as being $x \mathrm{mph}$ with some level of confidence. In the case of volume, if the observation period is increased from 5 minutes to 10 minutes, its value is expected to be twice as much. Data collected is not considered to be repeat observations of the same measure unless it is collected across several days for the same time duration under similar conditions. The standard deviation of volume counts can be used in assessing the stability of flow at a given location during a given time period across several days. The standard deviation of 15 -minute volume counts can be an indicator of how well a 15 -minute volume count represents the hourly flow rate, much like the commonly used peak hour factor (PHF).

Summary statistics for coefficient of variation (C.V.) values with and without outlier observations are presented in Table 2, along with the suggested values for further analysis.

TABLE 2 Coefficients of Variation of 15-minute Volume Counts for Arterials

|  | Coefficients of variation of 15 -minute volume counts (all data) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Volume (veh/hr/ln) | Average | Median | $70^{\text {th }}$ <br> Percentile | $80^{\text {th }}$ <br> Percentile |  |
| 0-200 | 0.92 | 0.93 | 0.97 | 1.00 |  |
| 200-400 | 0.25 | 0.24 | 0.28 | 0.29 |  |
| 400-600 | 0.16 | 0.15 | 0.17 | 0.18 |  |
| 600-800 | 0.13 | 0.13 | 0.13 | 0.14 |  |
| 800-1000 | 0.12 | 0.11 | 0.12 | 0.13 |  |
| 1000-1200 | 0.13 | 0.08 | 0.09 | 0.13 |  |
| 1200-1400 | 0.15 | 0.11 | 0.14 | 0.20 |  |
| 1400-1600 | 0.06 | 0.06 | 0.07 | 0.07 |  |
| 1600-1800 | 0.06 | 0.06 | 0.06 | 0.06 |  |
|  | Coefficients of variation of 15 -minute volume counts (w/o outliers) |  |  |  |  |
| Volume (veh/hr/ln) | Average | Median | $70^{\text {th }}$ <br> Percentile | $80^{\text {th }}$ <br> Percentile | Suggested Value |
| 0-200 | 0.93 | 0.93 | 0.98 | 1.01 | 1.00 |
| 200-400 | 0.23 | 0.23 | 0.25 | 0.27 | 0.26 |
| 400-600 | 0.16 | 0.15 | 0.17 | 0.18 | 0.18 |
| 600-800 | 0.13 | 0.13 | 0.13 | 0.14 | 0.14 |
| 800-1000 | 0.11 | 0.11 | 0.11 | 0.12 | 0.12 |
| 1000-1200 | 0.09 | 0.08 | 0.09 | 0.10 | 0.10 |
| 1200-1400 | 0.09 | 0.10 | 0.11 | 0.11 | 0.11 |
| 1400-1600 | 0.06 | 0.06 | 0.07 | 0.07 | 0.07 |
| 1600-1800 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |

Coefficient of variation values for arterial volume counts obtained after discarding outlier data are plotted in Figure 2.


FIGURE 2 Variability of arterial volume counts with hourly volume.
The coefficient of variation value for 15 -minute arterial volume count data is approximately equal to 1.0 for volumes less than $200 \mathrm{veh} / \mathrm{hr} / \mathrm{ln}$. After a sharp drop to 0.26 within the $200-400 \mathrm{veh} / \mathrm{hr} / \mathrm{ln}$ volume range, the C.V. value decreases smoothly as volume increases. Figure 2 suggests that below $400 \mathrm{veh} / \mathrm{hr} / \mathrm{ln}$ volume, the variability across $15-$ minute volume counts is a significant proportion of their average values. Therefore, caution should be exercised in extrapolating volume counts to hourly flow rate in this range, especially when fewer numbers of observations are available.

## Freeway

Traffic detector data was obtained for several freeways in the Washington, D.C. metropolitan area. Standard deviations for 5 -minute average speed and volume counts were calculated in a manner similar to that previously discussed for arterials.

## Speed

Summary statistics for standard deviation over several locations are plotted in Figure 3. The relationship between speed variability and hourly volume was again found to exhibit a U shaped curve. Few observations were recorded above $1800 \mathrm{veh} / \mathrm{hr} / \mathrm{ln}$ and the standard deviation of speed decreased in this range. This may be due to the fact that, after exhibiting
high speed variability during stop-and-go conditions just before the capacity limit, traffic again becomes uniform in near-jam conditions where there is little room for maneuvering.

Summary statistics for the standard deviation of speeds were calculated in the same manner as those for arterials. Results with and without outlier data are plotted in Figure 3. Suggested standard deviation values for each volume range are also displayed in Figure 3 (b).

(a) Variability of speed versus hourly volume plotted using all data.

(b) Variability of speed versus hourly volume plotted after discarding outlier data.

FIGURE 3 Variability of freeway speed with hourly volume.

Existing literature suggest a standard deviation value of 5 mph for spot speed on any highway type in any traffic area (1, 7). It can be seen from Figures 1 and 3 that this recommendation is suitable for only intermediate ranges of traffic volume on freeways. It is inadequate for arterials, in addition to both light and heavy traffic conditions on any roadway.

Since freeway data was collected at a small interval of 5-minutes, it was checked whether the characteristic $U$-shaped curve regenerates itself at a higher aggregation interval. Aggregation of speed for higher intervals was performed by taking both simple averages and weighting by volume. Standard deviation values of speed for these two methods matched up to the first decimal point for most volume groups. Variability of speed versus volume at different aggregation intervals is presented in Figure 4 for one location.


FIGURE 4 Variability of freeway speed at different aggregation intervals.

## Volume

Summary statistics for coefficient of variation (C.V.) values of freeway volume counts, with and without outlier observations, are presented in Table 3, along with the suggested values for further analysis.

TABLE 3 Coefficients of Variation of 5-minute Volume Counts for Freeways

|  | Coefficients of variation of 5-minute volume counts <br> (all data) |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Volume <br> (veh/hr/ln) | Average | Median | $\mathbf{7 0}^{\text {th }}$ <br> Percentile | $\mathbf{8 0}^{\text {th }}$ <br> Percentile |
| $0-200$ | 0.42 | 0.42 | 0.44 | 0.45 |
| $200-400$ | 0.28 | 0.27 | 0.29 | 0.31 |
| $400-600$ | 0.25 | 0.22 | 0.22 | 0.23 |
| $600-800$ | 0.18 | 0.14 | 0.17 | 0.22 |
| $800-1000$ | 0.16 | 0.13 | 0.18 | 0.22 |
| $1000-1200$ | 0.11 | 0.11 | 0.11 | 0.12 |
| $1200-1400$ | 0.11 | 0.11 | 0.12 | 0.13 |
| $1400-1600$ | 0.11 | 0.11 | 0.12 | 0.12 |
| $1600-1800$ | 0.09 | 0.09 | 0.11 | 0.11 |
| $1800-2000$ | 0.08 | 0.09 | 0.09 | 0.09 |
| $2000-2200$ | 0.09 | 0.09 | 0.09 | 0.09 |
| $2200-2400$ | 0.06 | 0.06 | 0.07 | 0.07 |


|  | Coefficients of variation of 5-minute volume counts <br> (w/o outliers) |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Volume <br> (veh/hr/ln) | Average | Median | $\mathbf{7 0}^{\text {th }}$ <br> Percentile | $\mathbf{8 0}^{\text {th }}$ <br> Percentile | Suggested <br> Value |
| $0-200$ | 0.42 | 0.42 | 0.44 | 0.45 | $\mathbf{0 . 4 5}$ |
| $200-400$ | 0.28 | 0.27 | 0.29 | 0.31 | $\mathbf{0 . 3 0}$ |
| $400-600$ | 0.20 | 0.21 | 0.22 | 0.22 | $\mathbf{0 . 2 2}$ |
| $600-800$ | 0.15 | 0.14 | 0.15 | 0.15 | $\mathbf{0 . 1 5}$ |
| $800-1000$ | 0.16 | 0.13 | 0.18 | 0.22 | $\mathbf{0 . 1 9}$ |
| $1000-1200$ | 0.11 | 0.11 | 0.11 | 0.12 | $\mathbf{0 . 1 2}$ |
| $1200-1400$ | 0.11 | 0.11 | 0.12 | 0.13 | $\mathbf{0 . 1 2}$ |
| $1400-1600$ | 0.11 | 0.11 | 0.12 | 0.12 | $\mathbf{0 . 1 2}$ |
| $1600-1800$ | 0.09 | 0.09 | 0.11 | 0.11 | $\mathbf{0 . 1 1}$ |
| $1800-2000$ | 0.08 | 0.09 | 0.09 | 0.09 | $\mathbf{0 . 0 9}$ |
| $2000-2200$ | 0.09 | 0.09 | 0.09 | 0.09 | $\mathbf{0 . 0 9}$ |
| $2200-2400$ | 0.06 | 0.06 | 0.07 | 0.07 | $\mathbf{0 . 0 7}$ |

Coefficient of variation values for arterial volume counts obtained after discarding outlier data are plotted in Figure 5.


FIGURE 5 Variability of freeway volume counts with hourly volume.
The standard deviation for 5-minute freeway volume counts start around 0.50 for volumes less than $200 \mathrm{veh} / \mathrm{hr} / \mathrm{ln}$ and steadily decrease with an increase in volume. Figure 5 suggests that below $400 \mathrm{veh} / \mathrm{hr} / \mathrm{ln}$ volume, the variability across 5 -minute volume counts is a significant proportion of the average values. Therefore, caution should be exercised in extrapolating volume counts to hourly flow rate in this range, especially when fewer numbers of observations are available

## SAMPLE SIZE CALCULATION

## Arterial

Speed
For sample size calculation, the Manual of Traffic Engineering Studies suggests a permitted error of $\pm 2.0$ to $\pm 4.0 \mathrm{mph}$ for traffic operations purposes (1). Suggested values for standard deviations which are displayed in Figure 1 (b) have been used in the sample size calculations. Eqn. $l$ has been used for the calculations, with the $t$-statistic replaced by the $z$-statistic, since standard deviation estimations are based on large sample sizes. Sample sizes for a permitted error of $\pm 4.0 \mathrm{mph}$ at $90 \%$ and $95 \%$ confidence levels (CL) are presented in Figure 6.


## FIGURE 6 Sample size estimates for arterial speed.

## Volume

Before calculating sample size for arterial volume counts, an idea about the likely hourly volume range for the study period should be obtained using historical data. Based on this hourly volume estimate, a conservative estimate of coefficient of variation can be selected from Table 2. Eqn. 5 can then be used to calculate sample size for a given percent accuracy and confidence level.

## Freeway

## Speed

Sample size calculations similar to arterials are performed, and values for a permitted error of $\pm 4.0 \mathrm{mph}$ at $90 \%$ and $95 \%$ confidence levels are presented in Figure 7.


FIGURE 7 Sample size estimates for freeway speed.

## Volume

The same procedure used for arterial volume can be used to calculate sample size for freeway volume counts. Coefficient of variation estimates for volume counts provided in Table 3 can be used for this purpose.

## CONCLUSIONS

There are several sample size estimation methods which avoid using an iterative procedure and the $t$-statistic for the sake of simplicity. Such methods are merely approximations of the fundamental sample size estimation method which uses standard deviation and the $t$-statistic. This fundamental method should be preferred whenever reliable estimates of the standard deviation can be obtained. This study estimated standard deviation values of speed and coefficient of variation values of volume counts based on statistical patterns of arterial and freeway traffic data. These estimates were based on a large number of observations at multiple sites, which justified the use of the $z$-statistic in the final calculation of sample size. Values obtained can be used for locations where data cannot be collected locally due to technical or economic limitations.

It has also been shown that the standard deviation value of 5 mph for spot speed suggested by existing literature is suitable only for freeways, not arterials. Moreover, a constant standard deviation value of 5 mph is not adequate for light and heavy traffic conditions on any roadway. This study also calculated the standard deviation of volume counts and suggested possible interpretation and usage, something which is not present in existing literature.

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## REFERENCES

1. Box, P.C., and J.C. Oppenlander. Manual of Traffic Engineering Studies, 4th ed., Institute of Transportation Engineers, Washington, D.C., USA, 1976.
2. Robertson, H.D. (Ed.). ITE Manual of Transportation Engineering Studies. Prentice Hall, Englewood Cliffs, N.J., 1994.
3. Oppenlander, J.C. Sample Size Determination for Travel Time and Delay Studies. Traffic Engineering, Vol. 46 No. 9, Institute of Traffic Engineers, Washington D.C., 1976, pp. 25-28.
4. Li, S., K. Zhu, B.H.W. van Gelder, J. Nagle, and C. Tuttle. Reconsideration of Sample Size Requirements for Field Traffic Data Collection Using GPS Devices. In Transportation Research Record: Journal of the Transportation Research Board, No. 1804, TRB, National Research Council, Washington, D.C., 2002, pp. 17-22.
5. Quiroga, C.A., and D. Bullock. Determination of Sample Sizes for Travel Time Studies. ITE Journal, Vol. 68, No. 8, Institute of Transportation Engineers, Washington, D.C., 1998, pp. 92-98.
6. Lomax, T., S. Turner, G. Shunk, H.S. Levinson, R.H. Pratt, P.N. Bay, and G.B. Douglas. Quantifying Congestion, Volume 2: User's Guide. NCHRP Report 398, TRB, National Research Council, Washington, D.C., 1997.
7. Oppenlander, J.C. Sample Size Determination for Spot-Speed Studies at Rural, Intermediate, and Urban Locations. In Highway Research Record: Journal of the Highway Research Board, No. 35, HRB, Washington, D.C., 1963, pp. 78-80.
8. Turner, S. M., and D.J. Holdener. Probe Vehicle Sample Sizes for Real-Time Information: The Houston Experience. In proceedings of the $6^{\text {th }}$ International Vehicle Navigation and Information Systems Conference, Seattle, Washington, 1995, pp. 3-10.
